Math 251

Assignment 7

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All exercises can be found in the book of Kenneth Rosen, seventh edition.

1.Why is f not a function from $\mathbb{R}$ to $\mathbb{R}$ if

(a) $f(x) = 1/x$?
(b) $f(x) = \sqrt{x}$?
(c) $f(x) = \pm\sqrt{x^2+1}$?

2.Which of the following functions $\mathbb{Z}$ to $\mathbb{Z}$ are onto:

(a) $f(n) = n - 1$
(b) $f(n) = n^2 + 1$
(c) $f(n) = n^3$.

3. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, is onto if

(a) $f(m, n) = 2m - n$.
(b) $f(m, n) = m^2 - n^2$.
(c) $f(m, n) = m$.
(d) $f(m, n) = |n|$.
(e) $f(m, n) = m - n$.

4. Prove that a strictly decreasing function from $\mathbb{R}$ to itself is one-one.

5. Show that the function $f(x) = |x|$ from the set of real numbers to the set of nonnegative real numbers is not invertible, but if the domain is restricted to the set of nonnegative real numbers, the resulting function is invertible.

6. Let $f(x) = 2x$ where the domain is the set of real numbers. What is

(a) $f(\mathbb{Z})$?
(b) $f(\mathbb{N})$?
(c) $f(\mathbb{R})$?

7. Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that

(a) $f(S \cup T) = f(S) \cup f(T)$.
(b) $f(S \cap T) \subseteq f(S) \cap f(T)$.

8. Let f be a function from the set A to the set B. Let S be a subset of B.We define the **inverse image** of S to be the subset of A whose elements are precisely all pre-images of all elements of S.We denote the inverse image of S by $f^{-1}(S)$, so

$$f^{-1}(S) = \{a \in A | f(a) \in S\}.$$

(Beware: The notation f^{-1} is used in two different ways. Do not confuse the notation introduced here with the notation $f^{-1}(y)$ for the value at y of the inverse of the invertible function f. Notice also that $f^{-1}(S)$, the inverse image of the set S, makes sense for all functions f, not just invertible functions.)

(i) If f be the function from A to B. Let S and T be sunsets of B. Show that

(a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$.
(b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.
(c) $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$

(ii) Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ define by $f(x) = x^2$. Find

(a) $f^{-1}(\{1\})$.
(b) $f^{-1}(\{x : 0 < x < 1\})$.
(c) $f^{-1}(\{x : x > 4\})$.

9. Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y. Show that the inverse of the composition $f \circ g$ is given by

$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}.$$

10. For each of these partial functions, determine its domain, codomain, domain of definition, and the set of values for which it is undefined. Also, determine whether it is a total function.

(a) $f : \mathbb{Z} \to \mathbb{R}$, $f(n) = 1/n$.
(b) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$, $f(m, n) = m/n$.
(c) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, $f(m, n) = mn$.
(e) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, $f(m, n) = m - n$ if $m > n$.